2.5 Continuity

2014-09-17 day 16

A function $f(x)$ is "continuous at $x=c$" if and only if [iff]

1) $f(x)$ is defined at $x=c$

2) $\lim_{x \to c} f(x)$ exists

All the work

3) $f(c) = \lim_{x \to c} f(x)$

this will often be easy b/c the fn will be a polynomial, a expon. log ⋮

2.5 Continuity

2014-09-17 day 16

$f(x) = \frac{x^2-4}{x-2}$; continuous at $x=2$?

discontinuous

a removable discontinuity

indeterminate $\frac{0}{0}$ undefined $\frac{\#}{0}$ means "not defined"

1) is $f(x)$ defined at $x=2$?

$f(2)$ would be $\frac{2^2-4}{2-2} = \frac{0}{0}$ uh oh.

No, the f^n is not continuous, b/c it is not defined at $x=2$.

2) $\lim_{x\to2} f(x) = \lim_{x\to2} \frac{x^2-4}{x-2} = \lim_{x\to2} x+2 = 4$

AHA! This function is not continuous, but

$$g(x) = \begin{cases} f(x), & x \neq 2 \\ 4, & x = 2 \end{cases}$$

would be continuous.

[because $g(2) = \lim_{x\to2} g(x)$]

2.5 Continuity

2014-09-17 day 16

$$g(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$$

continuous at $x=2$?

1) $g(x)$ defined? ✓

2) $\lim\limits_{x\to 2} g(x)$ exists.

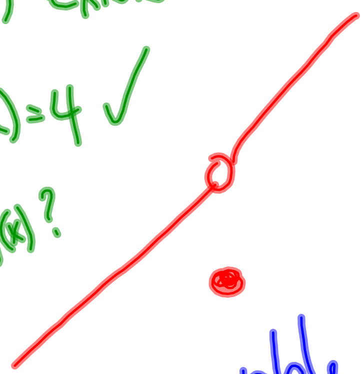
Yes! $\lim\limits_{x\to 2} g(x) = 4$ ✓

3) $g(2) = \lim\limits_{x\to 2} g(x)$?

No

fails (3)

$$h(x) = \begin{cases} \cancel{\dfrac{x^2-4}{x-2}}, & \\ 4, & x = 2 \end{cases}$$



a removable discontinuity

2.5 Continuity

2014-09-17 day 16

A function $f(x)$ is continuous on an open interval I iff

$f(x)$ is continuous at every x-value in I.

$(-\infty, \infty)$; $(-2, 4)$; $(0, 1)$

MUCH LESS important ideas

→ not ~~tested~~ on AP exam

→ provided to show types of decisions we can make.

A f: $f(x)$ is right-continuous at $x=c$ iff "continuous from the right"

1) def

2) $\lim_{x\to c^+} f(x)$ exists

3) $f(c) = \lim_{x\to c^+} f(x)$

$[1, 2)$

similarly, define "continuous from the left"

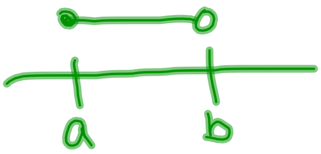
. . . $(1, 2]$ - - -

2.5 Continuity

2014-09-17 day 16

(a,b) : the interval containing all the x values between a and b, NOT including a, b

$a<x<b$



$[a,b)$: $a \leq x < b$

$(a,b]$: $a < x \leq b$

$[a,b]$: $a \leq x \leq b$

2.5 Continuity

2014-09-17 day 16

BAM Polynomials are continuous everywhere.

BAM f, g continuous ....

$\Longrightarrow$

* $f+g$ continuous ...
* $f-g$ continuous ...
* fg "
* $\frac{f}{g}$ ", whenever g is not 0.

BAM A rational f^n is continuous everywhere the denominator is nonzero

BAM [composition of f^ns]

If $\lim_{x \to c} g(x) = L$, and $f(x)$ is continuous at L, conditions

then $f(g(x))$ is continuous at c, and when

ok

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)$$

idea: limit symbol can be moved through a "f^n 'sign'" ('name' 'symbol')
provided limit of inside f^n exists, and
outside f^n is appropriately continuous.

COROLLARY: abs value of a cont. f^n is continuous.

COR-O-LARY:

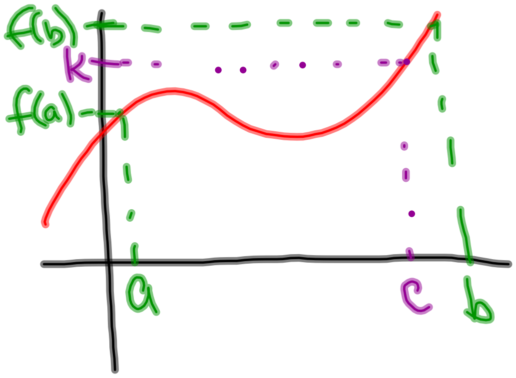
2.5 Continuity 2014-09-17 day 16

ALSO major milestone:

Intermediate Value Theorem (IVT):

If a function is continuous on [a, b], and k is a number in [f(a), f(b)] [or [f(b), f(a)]], then there exists a c in [a, b] with f(c) = k.

(Intermediate Value Theorem (IVT))



An existence theorem